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Peristaltic flow induced by a sinusoidal wave in a moving wall of a two-dimensional viscous fluid for moderately large Reynolds number is investigated. The boundary layer theory has been considered to be where its thickness is larger than the amplitude of the wavy wall. Solutions are obtained in terms of a series expansion with respect to a small amplitude ratio using a regular perturbation method. Velocity components, for both outer and inner flows for various values of the Reynolds number and wall velocity are represented graphically. The inner and outer velocity solutions are matched by a matching process. Certain interesting results regarding the axial and the transverse velocity components are discussed. This problem is regarded as an interesting application to mechanical engineering, where the possibility of fluid transportation without an external pressure is shown.

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I. INTRODUCTION

The study of the flow of fluids induced by the unsteady motion of a wall is of great practical importance in the field of biomechanics. Much attention has been paid to the propulsive mechanism of fish and bacteria in the field of biophysics. Gray [1] studied the drag on a swimming dolphin and found that this drag is much less than that on a solid body immersed in a fluid. Gray proposed a number of mechanisms which can reduce the drag, such as the effect of body shape (laminar aerofoil theory), the effect of flexible skin, and the unsteady motive effect. The latter one is related to the fluid mechanical developments concerning the swimming of fish and has raised a question on how an unsteady movement of a body immersed in a fluid induces a steady flow around it. The motive power of fish is mainly due to the flapping of the tail and fin, a waving motion of a body has thus an effect of thrusting the body, and this effect reduces the drag.

The flow of fluids induced by the sinusoidal wavy motion of a wall has been discussed by Taylor [5], Burns and Parkes [2], Dhar and Nandha [6], and Tanaka [4]. Tanaka studied the problem for both small and moderately large Reynolds numbers. While discussing the problem for moderately large Reynolds numbers, he observed that if the thickness of the boundary layer is larger than the wave amplitude, the technique employed for small

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Reynolds numbers can be applied to the case of moderately large Reynolds numbers as well.

The phenomenon of peristaltic transport has been of increasing interest to investigators in several engineering disciplines. From the mechanical point of view, peristalsis offers the opportunity of constructing pumps in which the transported medium does not come in direct contact with any moving parts, such as valves, plungers, and rotors. The mechanism of peristaltic transport has been also exploited for industrial applications such as sanitary fluid transport, blood pumps, heart lung machines, and the transport of corrosive fluids, where the contact of the fluid with the machinery parts is prohibited. To understand peristaltic action in various situations, several theoretical and experimental investigations have been made. Important contributions to the topic on Newtonian and non-Newtonian fluids include the studies of Fung and Yih [7], Yin and Fung [8], Takabatake and Ayukawa [9], Bohme and Friedrich [10], Siddiqui and Schwarz [11], Hakeem *et al.* [12], Mekheimer [13], Mekheimer and Abd elmaboud [14], Mekheimer *et al.* [15], Mekheimer *et al.* [16], and Abd elmaboud [17].

In the present paper we have studied the two dimensional flow of an incompressible fluid that is induced by a sinusoidal peristaltic wavy moving wall. Solutions are obtained in terms of a series expansion with respect to the small amplitude ratio by a regular perturbation method. The inner (boundary layer flow) and the outer (flow beyond the boundary layer) solutions are matched by a matching process given by Kevorkian and Cole [3]. Graphs of the velocity components, for both the outer and the inner flows, and for various values of the Reynolds numbers and wall velocity are drawn.

II. EQUATIONS OF MOTION

We consider a two-dimensional flow of an incompressible viscous fluid due to an infinite sinusoidal wavy wall moving with a constant velocity U and oscillating vertically with a frequency $\frac{c}{2\pi}$, x being the coordinate in the downstream direction of the flow, and y the coordinate perpendicular to it. The motion of the wall is described by

$$y = h(x, t) = a \cos \frac{2\pi}{\lambda}(x - ct), \quad (1)$$

where a is the amplitude of the wavy wall, λ is the wave length, and c is the wave speed. The equations of conservation of momentum for the fluid are

$$\nabla \cdot q = 0,$$

$$\rho \left(\frac{\partial q}{\partial t} + (q \cdot \nabla)q \right) = -\nabla p + \mu \nabla^2 q, \quad (2)$$

where p is the fluid pressure, ρ is the number density, and μ is the coefficient of viscosity. Here we assume that $\frac{a}{\lambda} \ll 1$. The boundary conditions are

$$u = \frac{(2\pi a)^{\frac{1}{2}}}{\lambda} U, \quad v = \frac{\partial h}{\partial t} \quad \text{at } y = h(x, t), \tag{3}$$

$$|u|, |v| < \infty \quad \text{as } y \rightarrow \infty.$$

We normalize all lengths by the characteristic length $\frac{\lambda}{2\pi}$, velocity components q by the characteristic speed c , the fluid pressure p by ρc^2 , and the time by the characteristic time $\frac{\lambda}{2\pi c}$. The above equations of motion of the fluid become

$$\frac{\partial q}{\partial t} + (q \cdot \nabla)q = -\nabla p + \frac{1}{R} \nabla^2 q, \tag{4}$$

where the Reynolds number $R = \frac{\lambda \rho c}{2\pi \mu}$, and the boundary conditions are

$$u = \varepsilon^{\frac{1}{2}} m U, \quad v = \frac{\partial h}{\partial t} \quad \text{at } y = h(x, t), \tag{5}$$

$$|u|, |v| < \infty \quad \text{as } y \rightarrow \infty,$$

where $h(x, t) = \varepsilon \cos(x - t)$, $m = (\frac{1}{\lambda})^{1/2}$, and $\varepsilon = \frac{2\pi a}{\lambda} \ll 1$.

By introducing the stream function $\psi(x, y, t)$ for the fluid. The governing equation (4), and the boundary conditions (5) are

$$\frac{\partial}{\partial t} \nabla^2 \psi + \frac{\partial \psi}{\partial x} \nabla^2 \frac{\partial \psi}{\partial y} - \frac{\partial \psi}{\partial y} \nabla^2 \frac{\partial \psi}{\partial x} = \frac{1}{R} \nabla^2 (\nabla^2 \psi),$$

$$\frac{\partial \psi}{\partial y} = \varepsilon^{\frac{1}{2}} m U, \quad -\frac{\partial \psi}{\partial x} = \frac{\partial h}{\partial t} \quad \text{at } y = h(x, t), \tag{6}$$

$$|\frac{\partial \psi}{\partial y}|, |\frac{\partial \psi}{\partial x}| < \infty \quad \text{as } y \rightarrow \infty.$$

III. SOLUTION OF THE PROBLEM

When Reynolds number becomes large, the boundary layer is formed. As we have assumed that the thickness of the boundary layer is larger than the wave amplitude, following Tanaka [4], a regular perturbation technique can be applied to the present problem. If δ is the thickness of the boundary layer, the non-dimensional may be defined as $\bar{y} = \frac{y}{\delta}$ and $\bar{\psi} = \frac{\psi}{\delta}$. When the viscous term is supposed to be of the same order as the inertia terms, we have that $\delta^2 R$ is $O(1)$, as usual. The boundary conditions at $y = h$ are expanded into a Taylor series around $h = 0$ in terms of the inner variables $\bar{\psi}$ and \bar{y} as

$$\frac{\partial \bar{\psi}}{\partial x}(0) + \frac{h}{\delta} \frac{\partial^2 \bar{\psi}}{\partial x \partial \bar{y}}(0) + \frac{1}{2} \frac{h^2}{\delta^2} \frac{\partial^3 \bar{\psi}}{\partial x \partial \bar{y}^2}(0) + \dots = -\frac{1}{\delta} \frac{\partial h}{\partial t},$$

$$\frac{\partial \bar{\psi}}{\partial \bar{y}}(0) + \frac{h}{\delta} \frac{\partial^2 \bar{\psi}}{\partial \bar{y}^2}(0) + \frac{1}{2} \frac{h^2}{\delta^2} \frac{\partial^3 \bar{\psi}}{\partial \bar{y}^3}(0) + \dots = \varepsilon^{\frac{1}{2}} m U. \tag{7}$$

In order that the Taylor series converges, $O(\delta)$ must be larger than $O(h)$, that is, $O(\varepsilon) < O(\delta)$. Following Tanaka [4], we take $\delta = r\varepsilon^{\frac{1}{2}}$, r being an arbitrary constant of $O(1)$. The outer flow (the flow beyond the boundary layer) is described by (6) in terms of the original variables (ψ, x, y, t) , while the inner flow (boundary layer flow) is described in terms of the inner variables $(\bar{\psi}, x, \bar{y}, t)$ on substituting $R = (r^2\varepsilon)^{-1}$ and $\delta = r\varepsilon^{\frac{1}{2}}$. As $\varepsilon \ll 1$, we can use a perturbation method and assume that the (outer flow) and (inner flow) can be expanded as a power series in $\varepsilon^{\frac{1}{2}}$ using

$$\psi = \sum_{n=1}^{\infty} \varepsilon^{\frac{n}{2}} \psi_n, \quad \bar{\psi} = \sum_{n=1}^{\infty} \varepsilon^{\frac{n}{2}} \bar{\psi}_n. \quad (8)$$

Substituting (8) and using $\bar{y} = (\frac{y}{\delta})$, $\bar{\psi} = (\frac{\psi}{\delta})$, $R = (r^2\varepsilon)$, $\delta = r\varepsilon^{\frac{1}{2}}$ in (6), and the boundary conditions (7) and then equating the coefficients of like power of $\varepsilon^{\frac{1}{2}}$. We obtain the equations and the boundary conditions corresponding to the first order and second order as follows.

First order ($[O(\varepsilon^{\frac{1}{2}})]$)

OUTER

$$\frac{\partial}{\partial t} \nabla^2 \psi_1 = 0. \quad (9)$$

INNER

$$\frac{\partial^4 \bar{\psi}_1}{\partial \bar{y}^4} - \frac{\partial^3 \bar{\psi}_1}{\partial t \partial \bar{y}^2} = 0, \quad (10)$$

$$\frac{\partial \bar{\psi}_1}{\partial \bar{y}}(0) = mU, \quad \frac{\partial \bar{\psi}_1}{\partial x}(0) = -\frac{\sin(x-t)}{r}. \quad (11)$$

Second order ($[O(\varepsilon)]$)

OUTER

$$\frac{\partial}{\partial t} \nabla^2 \psi_2 = \frac{\partial \psi_1}{\partial x} \nabla^2 \frac{\partial \psi_1}{\partial y} - \frac{\partial \psi_1}{\partial y} \nabla^2 \frac{\partial \psi_1}{\partial x}. \quad (12)$$

INNER

$$\frac{\partial^4 \bar{\psi}_2}{\partial \bar{y}^4} - \frac{\partial^3 \bar{\psi}_2}{\partial t \partial \bar{y}^2} = \frac{\partial \bar{\psi}_1}{\partial \bar{y}} \frac{\partial^3 \bar{\psi}_1}{\partial \bar{y}^2 \partial x} - \frac{\partial \bar{\psi}_1}{\partial x} \frac{\partial^3 \bar{\psi}_1}{\partial \bar{y}^3}, \quad (13)$$

$$\begin{aligned}\frac{\partial \bar{\psi}_2}{\partial \bar{y}}(0) &= -\frac{\cos(x-t)}{r} \frac{\partial^2 \bar{\psi}_1}{\partial \bar{y}^2}(0), \\ \frac{\partial \bar{\psi}_2}{\partial \bar{x}}(0) &= -\frac{\cos(x-t)}{r} \frac{\partial^2 \bar{\psi}_1}{\partial x \partial \bar{y}}(0),\end{aligned}\tag{14}$$

Third order ($[O(\varepsilon^{\frac{3}{2}})]$)

OUTER

$$\frac{\partial}{\partial t} \nabla^2 \psi_3 = r^2 \nabla^2 \nabla^2 \psi_1 - \frac{\partial \psi_1}{\partial y} \nabla^2 \frac{\partial \psi_2}{\partial x} - \frac{\partial \psi_2}{\partial y} \nabla^2 \frac{\partial \psi_1}{\partial x} + \frac{\partial \psi_1}{\partial x} \nabla^2 \frac{\partial \psi_2}{\partial y} + \frac{\partial \psi_2}{\partial x} \nabla^2 \frac{\partial \psi_1}{\partial y},\tag{15}$$

INNER

$$\begin{aligned}\frac{\partial^4 \bar{\psi}_3}{\partial \bar{y}^4} - \frac{\partial^3 \bar{\psi}_3}{\partial t \partial \bar{y}^2} &= -2r^2 \frac{\partial^4 \bar{\psi}_1}{\partial^2 x \partial^2 \bar{y}} + r^2 \frac{\partial^3 \bar{\psi}_1}{\partial t \partial x^2} \\ &+ \frac{\partial \bar{\psi}_1}{\partial \bar{y}} \frac{\partial^3 \bar{\psi}_2}{\partial \bar{y}^2 \partial x} + \frac{\partial \bar{\psi}_2}{\partial \bar{y}} \frac{\partial^3 \bar{\psi}_1}{\partial \bar{y}^2 \partial x} - \frac{\partial \bar{\psi}_1}{\partial \bar{x}} \frac{\partial^3 \bar{\psi}_2}{\partial \bar{y}^3} - \frac{\partial \bar{\psi}_2}{\partial \bar{x}} \frac{\partial^3 \bar{\psi}_1}{\partial \bar{y}^3},\end{aligned}\tag{16}$$

$$\begin{aligned}\frac{\partial \bar{\psi}_3}{\partial \bar{y}}(0) &= -\frac{1}{r} \cos(x-t) \frac{\partial^2 \bar{\psi}_2}{\partial \bar{y}^2}(0) - \frac{1}{2r^2} \cos^2(x-t) \frac{\partial^3 \bar{\psi}_1}{\partial \bar{y}^3}(0), \\ \frac{\partial \bar{\psi}_3}{\partial \bar{x}}(0) &= -\frac{1}{r} \cos(x-t) \frac{\partial^2 \bar{\psi}_2}{\partial x \partial \bar{y}}(0) - \frac{1}{2r^2} \cos^2(x-t) \frac{\partial^3 \bar{\psi}_1}{\partial x \partial \bar{y}^2}(0).\end{aligned}\tag{17}$$

Fourth order ($[O(\varepsilon^2)]$)

OUTER

$$\begin{aligned}\frac{\partial}{\partial t} \nabla^2 \psi_4 &= r^2 \nabla^2 \nabla^2 \psi_2 - \frac{\partial \psi_1}{\partial y} \nabla^2 \frac{\partial \psi_3}{\partial x} - \frac{\partial \psi_2}{\partial y} \nabla^2 \frac{\partial \psi_2}{\partial x} - \frac{\partial \psi_3}{\partial y} \nabla^2 \frac{\partial \psi_1}{\partial x} \\ &+ \frac{\partial \psi_1}{\partial x} \nabla^2 \frac{\partial \psi_3}{\partial y} + \frac{\partial \psi_2}{\partial x} \nabla^2 \frac{\partial \psi_2}{\partial y} + \frac{\partial \psi_3}{\partial x} \nabla^2 \frac{\partial \psi_1}{\partial y},\end{aligned}\tag{18}$$

INNER

$$\begin{aligned}\frac{\partial^4 \bar{\psi}_4}{\partial \bar{y}^4} - \frac{\partial^3 \bar{\psi}_4}{\partial t \partial \bar{y}^2} &= -2r^2 \frac{\partial^4 \bar{\psi}_2}{\partial^2 x \partial^2 \bar{y}} + r^2 \frac{\partial^3 \bar{\psi}_2}{\partial t \partial x^2} + r^2 \frac{\partial \bar{\psi}_1}{\partial \bar{y}} \frac{\partial^3 \bar{\psi}_1}{\partial x^3} - r^2 \frac{\partial \bar{\psi}_1}{\partial \bar{x}} \frac{\partial^3 \bar{\psi}_1}{\partial x^2 \partial \bar{y}} + \frac{\partial \bar{\psi}_1}{\partial \bar{y}} \frac{\partial^3 \bar{\psi}_3}{\partial \bar{y}^2 \partial x} \\ &+ \frac{\partial \bar{\psi}_2}{\partial \bar{y}} \frac{\partial^3 \bar{\psi}_2}{\partial \bar{y}^2 \partial x} + \frac{\partial \bar{\psi}_3}{\partial \bar{y}} \frac{\partial^3 \bar{\psi}_1}{\partial \bar{y}^2 \partial x} - \frac{\partial \bar{\psi}_1}{\partial \bar{x}} \frac{\partial^3 \bar{\psi}_3}{\partial \bar{y}^3} - \frac{\partial \bar{\psi}_2}{\partial \bar{x}} \frac{\partial^3 \bar{\psi}_2}{\partial \bar{y}^3} - \frac{\partial \bar{\psi}_3}{\partial \bar{x}} \frac{\partial^3 \bar{\psi}_1}{\partial \bar{y}^3},\end{aligned}\tag{19}$$

$$\begin{aligned} \frac{\partial \bar{\psi}_4}{\partial \bar{y}}(0) &= -\frac{1}{r} \cos(x-t) \frac{\partial^3 \bar{\psi}_3}{\partial \bar{y}^2}(0) - \frac{1}{2r^2} \cos^2(x-t) \frac{\partial^3 \bar{\psi}_2}{\partial \bar{y}^3}(0) - \frac{1}{6r^3} \cos^3(x-t) \frac{\partial^4 \bar{\psi}_1}{\partial \bar{y}^4}(0), \\ \frac{\partial \bar{\psi}_4}{\partial x}(0) &= -\frac{1}{r} \cos(x-t) \frac{\partial^2 \bar{\psi}_3}{\partial x \partial \bar{y}}(0) - \frac{1}{2r^2} \cos^2(x-t) \frac{\partial^3 \bar{\psi}_2}{\partial x \partial \bar{y}^2}(0) - \frac{1}{6r^3} \cos^3(x-t) \frac{\partial^4 \bar{\psi}_1}{\partial \bar{y}^3 \partial x}(0). \end{aligned} \tag{20}$$

A series of the inner solutions should satisfy the boundary conditions on the wall, while the outer solutions are only restricted to be bounded as y increases, but is

$$\left| \frac{\partial \psi_n}{\partial x} \right|, \quad \left| \frac{\partial \psi_n}{\partial y} \right| < \infty \text{ as } y \rightarrow \infty \text{ for } n = 1, 2, 3, \dots$$

It is necessary to match the outer and the inner solutions. Following Cole [3], the matching is carried out for both the x and y components of the velocity by the following principles:

$$\lim_{\varepsilon \rightarrow 0} \frac{1}{\varepsilon^{\frac{N}{2}}} \left[\sum_{n=1}^N \varepsilon^{\frac{n}{2}} \frac{\partial \psi_n}{\partial y} - \sum_{n=1}^N \varepsilon^{\frac{n}{2}} \frac{\partial \bar{\psi}_n}{\partial \bar{y}} \right] = 0, \tag{21}$$

$$\lim_{\varepsilon \rightarrow 0} \frac{1}{\varepsilon^{\frac{N}{2}}} \left[\sum_{n=1}^N \varepsilon^{\frac{n}{2}} \frac{\partial \psi_n}{\partial x} - r \varepsilon^{\frac{1}{2}} \sum_{n=1}^N \varepsilon^{\frac{n}{2}} \frac{\partial \bar{\psi}_n}{\partial x} \right] = 0, \tag{22}$$

where \bar{y} is fixed up to the N -th order of magnitude. Let us find first order solutions in the form:

$$\begin{aligned} \bar{\psi}_1(x, \bar{y}, t) &= F_1(\bar{y})e^{i(x-t)} + F_1^*(\bar{y})e^{-i(x-t)} + F_{1s}(\bar{y}), \\ \psi_1(x, y, t) &= f_1(y)e^{i(x-t)} + f_1^*e^{-i(x-t)} + f_{1s}(y). \end{aligned} \tag{23}$$

By substituting (23) in the first order differential equations (9) and (10) and the boundary conditions (11), we obtain the following system of equations:

$$\frac{d^4 F_1}{d\bar{y}^4} - i \frac{d^2 F_1}{d\bar{y}^2} = 0, \quad \frac{d^4 F_{1s}}{d\bar{y}^4} = 0, \quad \frac{d^2 f_1}{dy^2} - f_1 = 0, \quad \frac{d^4 F_{1s}}{d\bar{y}^4} = 0, \tag{24}$$

and their solutions

$$\begin{aligned} F_1 &= A_1 e^{-\lambda \bar{y}} + \lambda A_1 \bar{y} - A_1 + \frac{1}{2r}, \\ \frac{dF_{1s}}{d\bar{y}} &= B_1 \bar{y}^2 + B_2 \bar{y} + mU, \\ f_1 &= ae^{-y}. \end{aligned} \tag{25}$$

Following Tanaka [4] $\frac{df_{1s}}{dy} = C_1,$

where $\lambda = \sqrt{-i}$ and A, B, a are constants. Substituting (25) into (21), we have

$$\begin{aligned} & \lim_{\varepsilon \rightarrow 0} \frac{1}{\varepsilon^{\frac{1}{2}}} \left[\varepsilon^{\frac{1}{2}} \frac{\partial \psi_1}{\partial y} - \varepsilon^{\frac{1}{2}} \frac{\partial \bar{\psi}_1}{\partial \bar{y}} \right] \\ &= \lim_{\varepsilon \rightarrow 0} \left[-ae^{-y} e^{i(x-t)} + \text{c.c.} + \frac{df_{1s}}{dy} - (-A_1 \lambda e^{-\lambda \bar{y}} + \lambda A_1) - B_1 \bar{y}^2 - B_2 \bar{y} - mU \right] = 0. \end{aligned}$$

Where c.c. stands for the corresponding complex conjugate. Taking account that $y = r\varepsilon^{\frac{1}{2}}\bar{y}$, expanding the exponential as

$$e^{-y} = e^{-r\varepsilon^{\frac{1}{2}}\bar{y}} = 1 - r\varepsilon^{\frac{1}{2}}\bar{y} + r^2\varepsilon\bar{y}^2 + \dots,$$

and noting that $\exp(-\lambda\bar{y}) = \exp(-\lambda y/r\varepsilon^{\frac{1}{2}})$ decays very rapidly as $\varepsilon \rightarrow 0$ (which is called transcendentally small (T.S.T) and is neglected in the matching process), we have

$$\lim_{\varepsilon \rightarrow 0} [(-a - \lambda A_1) + \text{c.c.} + C_1 - B_1 \bar{y}^2 + B_2 \bar{y} - mU + T.S.T] = 0.$$

When a similar process is carried out for (22), we get

$$\lim_{\varepsilon \rightarrow 0} \frac{1}{\varepsilon^{\frac{1}{2}}} \left[\varepsilon^{\frac{1}{2}} \frac{\partial \psi_1}{\partial x} - r\varepsilon^{\frac{1}{2}} \frac{\partial \bar{\psi}_1}{\partial x} \right] = \lim_{\varepsilon \rightarrow 0} \frac{1}{\varepsilon^{\frac{1}{2}}} \left[\varepsilon^{\frac{1}{2}} (ia) e^{i(x-t)} + \text{c.c.} + o(\varepsilon) \right] = 0,$$

so that matching condition is satisfied $a = 0$. Thus we have

$$A_1 = B_1 = B_2 = 0, \quad C_1 = mU$$

and the first order solutions are obtained as

$$\begin{aligned} \psi_1 &= mUy, \\ \bar{\psi}_1 &= \frac{1}{2r} e^{i(x-t)} + \frac{1}{2r} e^{-i(x-t)} + mU\bar{y}. \end{aligned} \tag{26}$$

Next we seek the solutions $\psi_2, \bar{\psi}_2$ in the following form:

$$\begin{aligned} \bar{\psi}_2(x, \bar{y}, t) &= F_2(\bar{y}) e^{2i(x-t)} + F_{21}(\bar{y}) e^{i(x-t)} + \text{c.c.} + F_{2s}(\bar{y}), \\ \psi_2(x, y, t) &= f_1(y) e^{2i(x-t)} + f_{21} e^{i(x-t)} + \text{c.c.} + f_{2s}(y). \end{aligned} \tag{27}$$

Substituting (27) and (26) into (12)-(14) we get, after some calculations,

$$\begin{aligned} \bar{\psi}_2 &= \left(-\frac{i\lambda}{2} e^{-\lambda \bar{y}} - \frac{\bar{y}}{2} + \frac{i\lambda}{2} \right) e^{i(x-t)} + \text{c.c.}, \\ \psi_2 &= \frac{1}{2} e^{-y} e^{i(x-t)} + \text{c.c.} \end{aligned} \tag{28}$$

Let us now seek third order solutions in the form

$$\begin{aligned} \bar{\psi}_3(x, \bar{y}, t) &= F_3(\bar{y}) e^{3i(x-t)} + F_{32}(\bar{y}) e^{2i(x-t)} + F_{31}(\bar{y}) e^{i(x-t)} + \text{c.c.} + F_{3s}(\bar{y}), \\ \psi_3(x, y, t) &= f_3(y) e^{3i(x-t)} + f_{31} e^{2i(x-t)} + f_{32}(y) e^{i(x-t)} + \text{c.c.} + f_{3s}(y), \end{aligned} \tag{29}$$

where

$$\begin{aligned}
 F_3 &= 0, \\
 F_{32} &= \frac{1}{4r} - \frac{1}{4r}e^{-\lambda\bar{y}}, \\
 F_{31} &= \left(\frac{-imU\lambda}{4} - \frac{mU\bar{y}}{4} - \frac{ir}{2} \right) e^{-\lambda\bar{y}} - \frac{i\lambda r\bar{y}}{2} + \frac{r\bar{y}^2}{4} + \frac{ir}{2} + \frac{imU\lambda}{4}, \\
 f_{31} &= \frac{ir\lambda}{2}e^{-y}, \\
 \frac{dF_{3S}}{d\bar{y}} &= \frac{\lambda}{4r}e^{-\lambda\bar{y}} + \frac{\lambda^*}{4r}e^{-\lambda^*\bar{y}}.
 \end{aligned} \tag{30}$$

We shall now seek the fourth order solutions in the following form:

$$\begin{aligned}
 \bar{\psi}_4(x, \bar{y}, t) &= F_4(\bar{y})e^{4i(x-t)} + F_{43}(\bar{y})e^{3i(x-t)} + F_{42}(\bar{y})e^{2i(x-t)} + F_{41}(\bar{y})e^{i(x-t)} + \text{c.c.} + F_{3s}(\bar{y}), \\
 \psi_4(x, y, t) &= f_4(y)e^{4i(x-t)} + f_{43}(y)e^{3i(x-t)} + f_{42}(y)e^{2i(x-t)} + f_{41}(y)e^{i(x-t)} + \text{c.c.} + f_{3s}(y),
 \end{aligned} \tag{31}$$

where

$$\begin{aligned}
 F_4 &= 0, \\
 F_{43} &= \frac{-\lambda}{16r^2}e^{-\lambda\bar{y}}, \\
 F_{42} &= \frac{mU}{4(2^{\frac{1}{2}})r}e^{(2)^{\frac{1}{2}}\lambda\bar{y}} - \left(\frac{mU}{4r} + \frac{i\lambda}{4} + \frac{mU\lambda\bar{y}}{8r} - \frac{\bar{y}}{4} \right) e^{-\lambda\bar{y}} - \frac{\bar{y}}{2} - \frac{mU}{4(2^{\frac{1}{2}})r} + \frac{mU}{4r} + \frac{i\lambda}{4}, \\
 F_{41} &= \left(-\frac{\lambda}{8r^2} + \frac{r^2\lambda}{4} - \frac{3i\lambda m^2 U^2 r}{16} - \frac{imUr}{4} \right) e^{-\lambda\bar{y}} \\
 &\quad + \left(\frac{ir^2\bar{y}}{4} - \frac{3m^2 U^2 \bar{y}}{16} - \frac{i\lambda mUr\bar{y}}{4} - \frac{\lambda m^2 U^2 \bar{y}^2}{16} \right) e^{-\lambda\bar{y}} \\
 &\quad - \frac{\lambda^*}{16r^2}e^{-\lambda^*\bar{y}} - \frac{r^2\bar{y}^3}{12} + \frac{ir^2\lambda\bar{y}^2}{4} - \frac{ir^2\bar{y}}{2} - \frac{r^2\lambda}{4} + \frac{3im^2 U^2 \lambda}{16} + \frac{imUr}{4}, \\
 \frac{dF_{4S}}{d\bar{y}} &= \frac{1}{4}e^{-(\lambda+\lambda^*)\bar{y}} - \left(-\frac{\lambda mU}{8r} + \frac{imU\bar{y}}{8r} + \frac{\lambda\bar{y}}{4} + \frac{i}{4} + \frac{1}{4} \right) e^{-\lambda\bar{y}} \\
 &\quad - \left(-\frac{\lambda^* mU}{8r} - \frac{imU\bar{y}}{8r} + \frac{\lambda^*\bar{y}}{4} - \frac{i}{4} + \frac{1}{4} \right) e^{-\lambda^*\bar{y}} + \frac{1}{4} - \frac{mU\lambda}{4r} - \frac{mU\lambda^*}{4r}, \\
 f_{42} &= \frac{1}{4}e^{-2y}, \\
 f_{41} &= \frac{ir^2}{2}e^{-y}.
 \end{aligned} \tag{32}$$

In a similar way higher order solutions can also be found. But due to the complexities involved in the problem, we are terminating our analysis with a fourth order solution.

IV. RESULTS AND DISCUSSIONS

For the flow induced by a sinusoidal peristaltic wavy moving wall, we found that the first order and the fourth order solutions consist of the steady part in addition to the periodic one. But the contribution of the steady term in the fourth order solution is more significant to the solution. So we shall take up for discussion the first and fourth order solutions. The velocity components of the fluid for the outer and inner flows have been plotted against y and $\bar{y} = \frac{y}{\delta}$, where δ the thickness of the boundary layer, respectively, for various values of Reynolds number R , the velocity of the wall U and $x - t$ at $\varepsilon = 0.1$.

The behavior of the axial velocity component of the inner flow U_i can be studied from Figs. 1 and 2. We find that U_i for $x - t = 0$ is greater than that of $x - t = \pi$. The effect of the increase in the Reynolds number is to increase the magnitude of the flow velocity for $x - t = 0$ and $x - t = \pi$, while a reverse direction has occurred for $x - t = \pi$, also it is interesting to note that U_i is oscillating between positive and negative values.

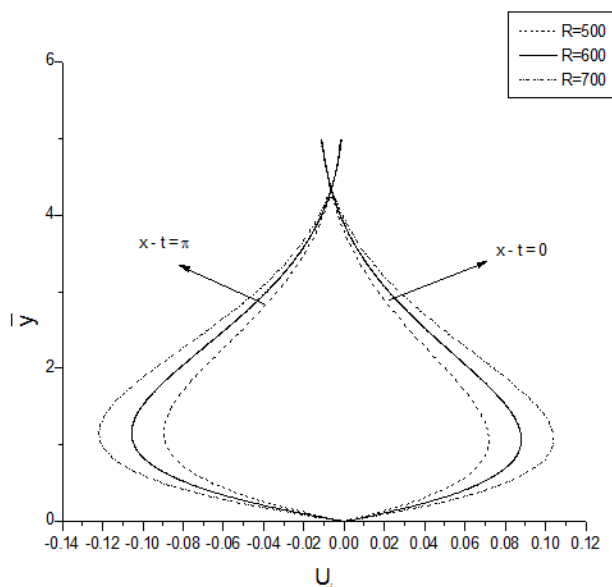


FIG. 1: Axial velocity component of the fluid U_i in the boundary layer for $\varepsilon = 0.1$, $U = 0$, and $m = 0.316$.

From Fig. 2, we observe that the axial velocity component for the inner flow increases as the velocity of the moving wall U increases for $x - t = 0$ and for $x - t = \pi$. However, the effects of the moving wall makes the profiles of the fluid more separated.

Figures 3 and 4 describe the nature of the transverse velocity component of the fluid V_i of the inner flow. We notice from Fig. 3 that an increase in the Reynolds number causes the transverse velocity to be oscillatory, it increases its magnitude for both $x - t = 0$ and $x - t = \pi$, but reverse its direction for $x - t = \pi$. For both cases $x - t = 0$ and $x - t = \pi$, we

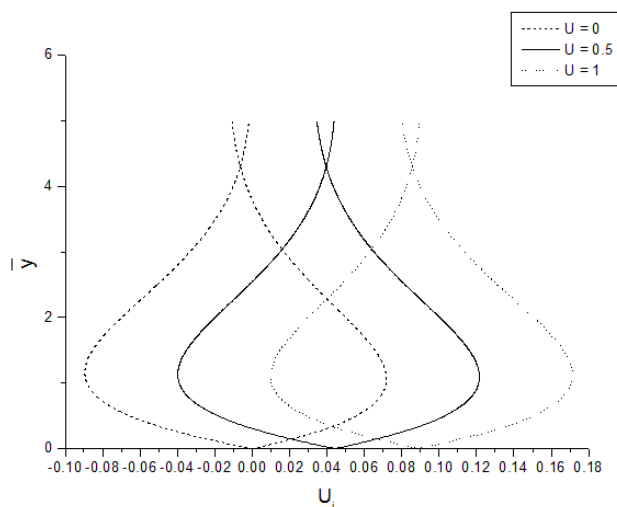


FIG. 2: Axial velocity component of the fluid U_i in the boundary layer for $\varepsilon = 0.1$, $R = 500$, and $m = 0.316$.

found that V_i decreases as y increases and approach more or less the same constant value. In Fig. 4, we observe that an increase of the velocity U of the moving wall will increase the transverse velocity of the inner flow, and this kind of motion of the wall makes the profiles of V_i much closer. Also, we see that for both cases of oscillations $x - t = 0$ and $x - t = \pi$, the transverse velocity V_i always move forward in the positive x -direction as the velocity U of the wall increases. Moreover, from these figures we see initially some oscillatory nature in the fluid, but it becomes steady as \bar{y} increases.

Figures 5 and 6 describe the behavior of the inner steady streaming parts of the fluid against \bar{y} for various values of U . Fig. 5 shows that the steady axial velocity component of the inner flow U_{is} increases for small values of the velocity U of the moving wall, and the profile of the velocity curves are more separated. It is also interesting to show that the inner steady velocity flow approaches a constant value in the form of the damped oscillation with respect to the distance from the wall, as shown in Figs. 5 and 6. However, we can say that the progressive motion of the wall causes, at first, the periodic flow in the boundary layer to have the same phase as that of the wall motion, then it causes flows of higher harmony in the boundary layer, inducing the periodic flow in the outer layer successively. At first the steady flow was induced only in the boundary layer, and after that there was an influence upon the outer flow, giving rise to the steady flow in the whole region of the field.

Also, we noticed that at large value of the Reynolds number the steady flow velocity, U_{is}/ε^2 , approaches to 0.25 away from the wall, which coincides with results in [4].

In Fig. 7 we study the nature of the transverse velocity component of the outer flow V_o . It is shown that an increase in the Reynolds number R , increases the magnitude of V_o for both $x - t = \pi$ and $x - t = 0$, and for both cases V_o becomes steady as y increases.

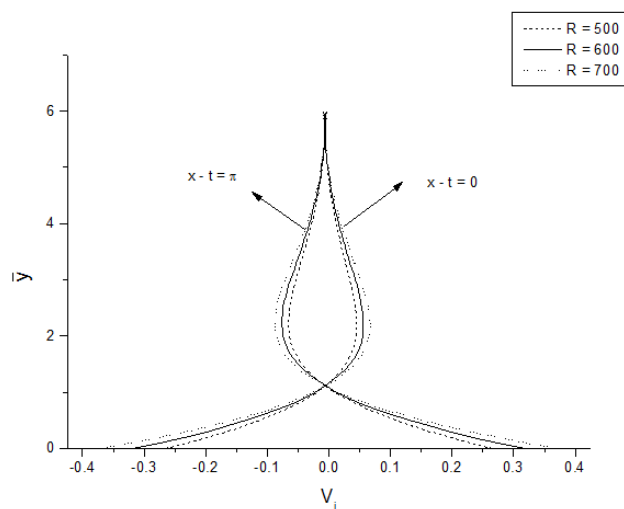


FIG. 3: Transverse velocity component of the fluid V_i in the boundary layer for $\varepsilon = 0.1$, $U = 0$, and $m = 0.316$.

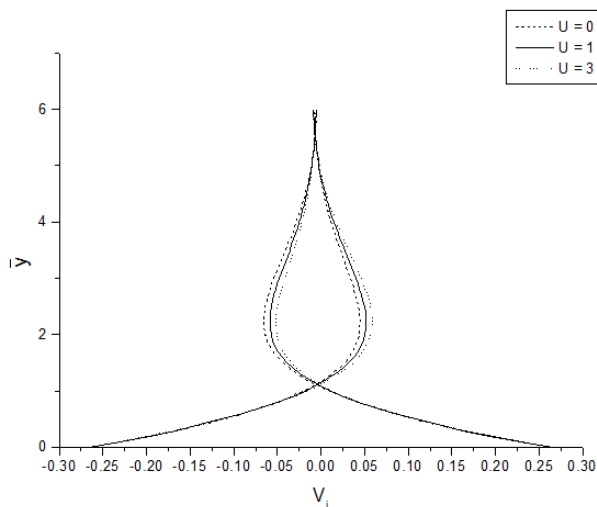


FIG. 4: Transverse velocity component of the fluid V_i in the boundary layer for $\varepsilon = 0.1$, $R = 500$, and $m = 0.316$.

Moreover, the transverse outer velocity becomes negative for $x - t = 0$, while it takes positive values for $x - t = \pi$.

Fig. 8 elucidates the behavior of the axial velocity component U_o of the fluid of the outer flow. We observe that as the values of U increase, the axial velocity of the outer flow

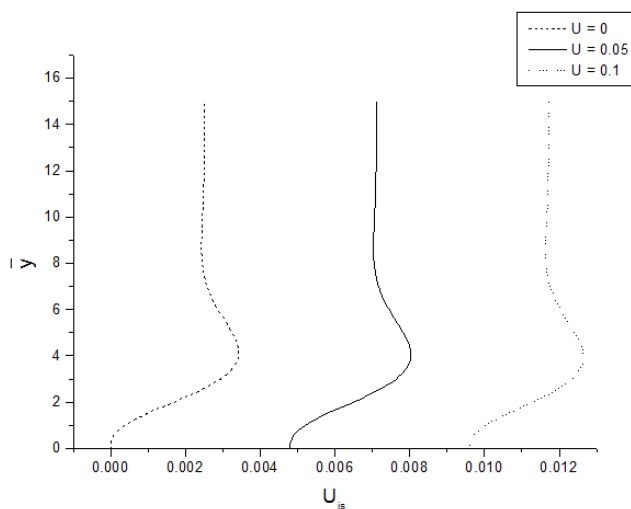


FIG. 5: Induced steady axial velocity component of the fluid U_{is} in the boundary layer for $\varepsilon = 0.1$, $R = 500$, and $m = 0.316$.

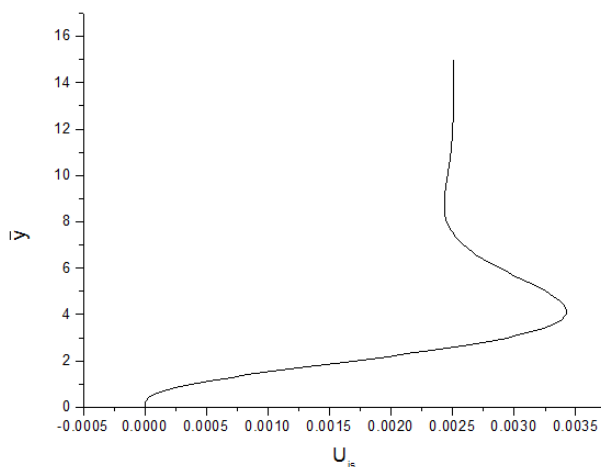


FIG. 6: Induced steady axial velocity component of the fluid U_{is} in the boundary layer for different values of Reynolds number at $\varepsilon = 0.1$, $U = 0$, and $m = 0.316$.

increases. Also, we note that the axial velocity increases as y increases for $x - t = 0$, but its behavior is reversed for $x - t = \pi$. It is interesting to notice the steadiness of U_o as y increases for both $x - t = 0$ and $x - t = \pi$. Further, it approaches almost equal values. Finally, the axial velocity U_o of the outer flow has approximately the same value at any

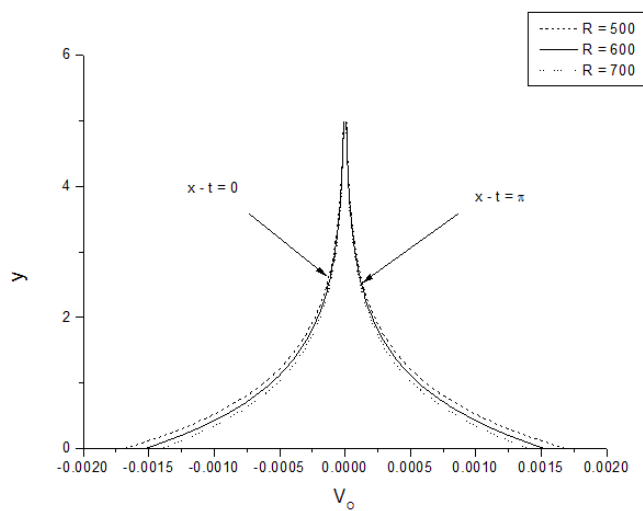


FIG. 7: Outer flow transverse velocity component of the fluid V_o for different values of the Reynolds number, taking $\varepsilon = 0.1$.

value of the Reynolds number R , as shown in Fig. 9.

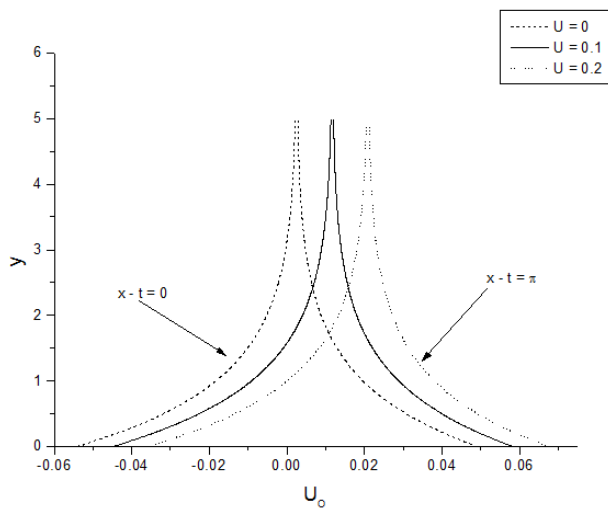


FIG. 8: Outer flow axial velocity component of the fluid U_o for $\varepsilon = 0.1$, $R = 500$, and $m = 0.316$.

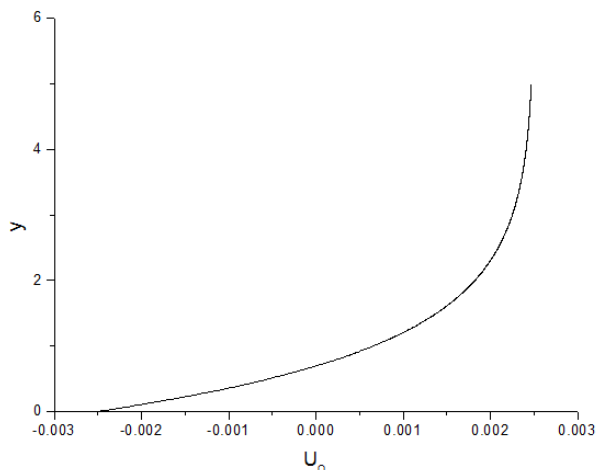


FIG. 9: Outer flow axial velocity component of the fluid U_0 for different values of the Reynolds number at $\varepsilon = 0.1$, $U = 0$, and $m = 0.316$.

V. CONCLUSION

The effect of a normal oscillation of a wavy moving wall on the induced flow of a two-dimensional viscous fluid is investigated on the basis of boundary layer theory in the case where the thickness of the boundary layer is larger than the amplitude of the wavy wall. The velocity components of the fluid of the outer and inner flow are obtained in terms of a series expansion with respect to small amplitude by a regular perturbation method. The inner and outer solutions are matched by the matching process. Graphs of the velocity components, both for outer flow and inner flow for various values of R and U are drawn. The main finding can be summarized as follows:

The steady streaming flow can be induced due to nonlinearity by the progressive wavy motion of the wall and is proportional to ε^2 .

The fluid on a progressing wavy wall is transported in the direction of the wave propagation. The axial steady flow velocity U_{is}/ε^2 approaches to 0.25 away from the wall, which coincides with the results of Tanaka [4].

* The velocity components for the inner and outer flows are increased as the wall velocity U increases.

* The study of this problem is very applicable on the basic mechanism of the swimming of fishes and to mechanical engineering, where there is a possibility of the fluid transportation without an external pressure.

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